Assignment 2: Differentiation Techniques
(*80 marks total*)

Your mark will be given as a percentage.

[*10 marks*] 1. Find the derivative in each case. You need not simplify your answer.

(5 marks) a. $f(t) = (-3t^2 + \frac{1}{\sqrt[3]{4t}})(t^3 + 2\sqrt[4]{t})$

(5 marks) b. $g(t) = \frac{\sqrt{t}+4}{\sqrt[3]{t}-5}$.

[*10 marks*] 2. Find the derivative in each case. Simplify your answer.

(5 marks) a. $f(x) = (3x^2 - 1)^4(5 - 2x)^6$.

(5 marks) b. $f(x) = \sqrt[3]{\frac{2x-5}{3x-2}}$.

[*10 marks*] 3. Find an equation of the tangent line to the graph of:

(5 marks) a. $f(x) = \sqrt{2x^3 + 7x}$ at $x = 1$.

(5 marks) b. $g(x) = \left(\frac{2x+3}{x-1}\right)^3$ at $x = 2$

[*10 marks*] 4. For the function $f(x) = \frac{2}{3}x^3 - 7x^2 + 24x + 11$,

(6 marks) a. Give all the points where the tangent line is horizontal. Note a point is given by two co-ordinates (_, _). Give exact values.

(4 marks) b. Give an equation of the tangent line at each of these points. Give exact values.

[*10 marks*] 5. Find $y^{(4)}(x)$ for $y = \frac{2}{x^3} - \frac{5}{\sqrt{x}}$

[*10 marks*]   6.   Suppose that the number of items sold after t months is modelled by

$$N(t) = \frac{500t}{(25t + 2)^2}$$

(3 marks)   a. Find $N'(6)$ and $N'(12)$.

(3 marks)   b. Find $N''(6)$, $N''(12)$.

(4 marks)   c. What do the answers in a. and b. tell you about $N(t)$?

[*10 marks*]   7.   Let $f(x) = \frac{x+1}{x-1}$ and $g(x) = \sqrt{x}$. Let $g(x) = \sqrt{x}$ and let

$h(x) = f \circ g$.

(6 marks)   a. Find $h'(x)$

(4 marks)   b. Find $h'(4)$

[*10 marks*]   8.   A company determines the cost, in thousands of dollars for producing x items is given by $C(x) = \sqrt{500x^2 - x + 75}$.

Suppose that it plans to boost production in t months from now according to the function $x(t) = 30t + 7$. Use implicit differentiation to find how fast the cost will be rising 6 months from now.